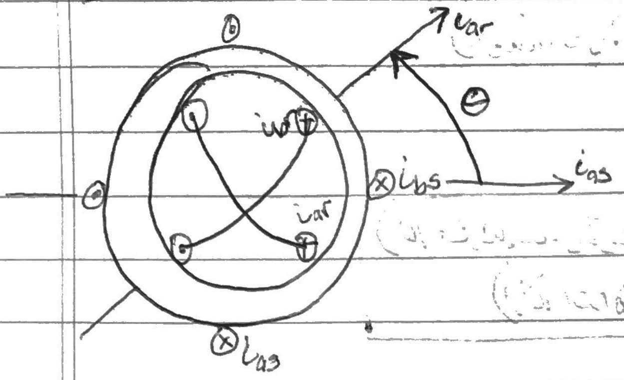
Induction Machines



* Same equations for flux linkage as synchronous machines
* Short the windings on the rotor

* Recall, the stator windings in 2 (and 3) phase produce a rotating magnetic field
  $H_s = H_{so}\cos(\omega_s t - \theta)$
  This is key!

1) Rotating (changing) magnetic field
2) By Faraday's law, this induces a changing electric field on rotor
3) The changing electric field produces a changing current on the rotor
4) By Lenz's law, the current flows in a direction that creates a magnetic field opposing the change in magnetic flux
5) The interaction between the stator and rotor magnetic fields create a torque on the rotor, causing it to rotate in the direction of the changing magnetic field

* AC field on rotor: $\omega_r \neq 0$

Frequency condition: $\omega_m = \omega_s - \omega_r \Rightarrow \omega_r = \omega_s - \omega_m$

Slip: $s = \dfrac{\omega_s - \omega_m}{\omega_s} \Rightarrow \omega_r = s\omega_s$

$\omega_m = \omega_s - s\omega_s \Rightarrow \omega_m = (1-s)\omega_s$

Assume: $i_{ar} = I_r \cos(s\omega_s t + \beta)$ $\quad i_{as} = I_s \cos(\omega_s t)$ $\quad \theta = (1-s)\omega_s t + \gamma$

$i_{br} = I_r \sin(s\omega_s t + \beta)$ $\quad i_{bs} = I_s \sin(\omega_s t)$

Voltage: $V_{as} = i_{as} R_s + \frac{d\lambda_{as}}{dt}$

$$\boxed{\begin{aligned} V_s \cos(\omega_s t + \theta_v) &= R_s I_s \cos(\omega_s t) + \omega_s L_s I_s \cos(\omega_s t + 90^\circ) \\ &+ \omega_s M I_r \cos(\omega_s t + (\gamma + \beta + 90^\circ)) \end{aligned}}$$

$V_{ar} = i_{ar} R_r + \frac{d\lambda_{ar}}{dt}$

$$\boxed{\begin{aligned} V_r \cos(s\omega_s t + \theta_r) &= R_r I_r \cos(s\omega_s t + \beta) + s\omega_s L_r I_r \cos(s\omega_s t + (\beta + 90^\circ)) \\ &+ s\omega_s M I_s \cos(s\omega_s t + (-\gamma + 90^\circ)) \end{aligned}}$$

As Phasors: $$\boxed{\frac{V_s}{\sqrt{2}} \angle \theta_v = R_s\left(\frac{I_s}{\sqrt{2}} \angle 0\right) + j\omega_s L_s\left(\frac{I_s}{\sqrt{2}} \angle 0\right) + j\omega_s M\left(\frac{I_r}{\sqrt{2}} \angle \gamma + \beta\right)}$$

$$\frac{V_r}{\sqrt{2}} \angle \theta_r = R_r\left(\frac{I_r}{\sqrt{2}} \angle \beta\right) + js\omega_s L_r\left(\frac{I_r}{\sqrt{2}} \angle \beta\right) + js\omega_s M\left(\frac{I_s}{\sqrt{2}} \angle -\gamma\right)$$

$$\rightarrow \boxed{\frac{V_r}{s\sqrt{2}} \angle \theta_r + \gamma = \frac{R_r}{s}\left(\frac{I_r}{\sqrt{2}} \angle \beta + \gamma\right) + j\omega_s L_r\left(\frac{I_r}{\sqrt{2}} \angle \beta + \gamma\right) + j\omega_s M\left(\frac{I_s}{\sqrt{2}} \angle 0\right)}$$

Recall: $M = \frac{\mu_0 \pi r l N_s N_r}{2g}$

*use this to define

$L_s = L_{ls} + \left(\frac{N_s}{N_r}\right) M$

$L_r = L_{lr} + \left(\frac{N_r}{N_s}\right) M$

$$\boxed{\begin{aligned}\frac{V_s}{\sqrt{2}}\angle\theta_v &= R_s\left(\frac{I_s}{\sqrt{2}}\angle 0\right) + j\omega_s L_{ls}\left(\frac{I_s}{\sqrt{2}}\angle 0\right) + j\omega_s M\left(\frac{N_s}{N_r}\right)\left(\frac{I_s}{\sqrt{2}}\angle 0\right) \\ &\quad + j\omega_s M\left(\frac{I_r}{\sqrt{2}}\angle\gamma+\beta\right)\end{aligned}}$$

$$\boxed{\begin{aligned}\frac{V_r\angle\theta_r+\gamma}{s\sqrt{2}} &= \frac{R_r}{s}\left(\frac{I_r}{\sqrt{2}}\angle\beta+\gamma\right) + j\omega_s L_{lr}\left(\frac{I_r}{\sqrt{2}}\angle\beta+\gamma\right) + j\omega_s M\left(\frac{N_r}{N_s}\right)\left(\frac{I_r}{\sqrt{2}}\angle\beta+\gamma\right) \\ &\quad + j\omega_s M\left(\frac{I_s}{\sqrt{2}}\angle 0\right)\end{aligned}}$$

Define Rotor Values referenced to the Stator:

$$I_r' = I_r\left(\frac{N_r}{N_s}\right) \qquad V_r' = \left(\frac{N_s}{N_r}\right)V_r$$

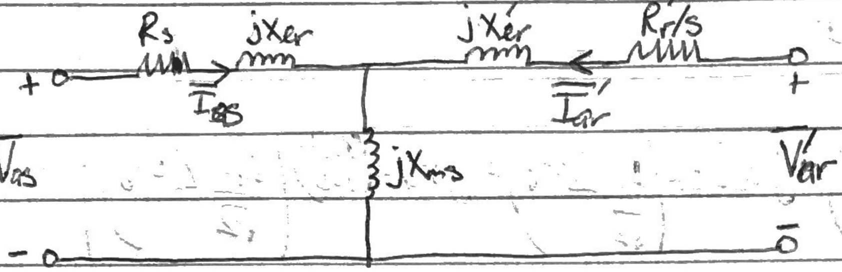
$$R_r' = R_r\left(\frac{N_s}{N_r}\right)^2 \qquad L_{lr}' = L_{lr}\left(\frac{N_s}{N_r}\right)^2$$

$$\boxed{\begin{aligned}\frac{V_s}{\sqrt{2}}\angle\theta_v &= R_s\left(\frac{I_s}{\sqrt{2}}\angle 0\right) + j\omega_s L_{ls}\left(\frac{I_s}{\sqrt{2}}\angle 0\right) + j\omega_s M\left(\frac{N_s}{N_r}\right)\left(\frac{I_s}{\sqrt{2}}\angle 0\right) \\ &\quad + j\omega_s M\left(\frac{N_s}{N_r}\right)\left(\frac{I_r'}{\sqrt{2}}\angle\gamma+\beta\right) \\[1em] \frac{V_r'}{s\sqrt{2}}\angle\theta_r+\gamma &= \frac{R_r'}{s}\left(\frac{I_r'}{\sqrt{2}}\angle\beta+\gamma\right) + j\omega_s L_{lr}'\left(\frac{I_r'}{\sqrt{2}}\angle\beta+\gamma\right) \\ &\quad + j\omega_s M\left(\frac{N_s}{N_r}\right)\left(\frac{I_r'}{\sqrt{2}}\angle\beta+\gamma\right) + j\omega_s M\left(\frac{N_s}{N_r}\right)\left(\frac{I_s}{\sqrt{2}}\angle 0\right)\end{aligned}}$$

<u>As a circuit:</u> $X_{ls} = \omega_s L_{ls}$

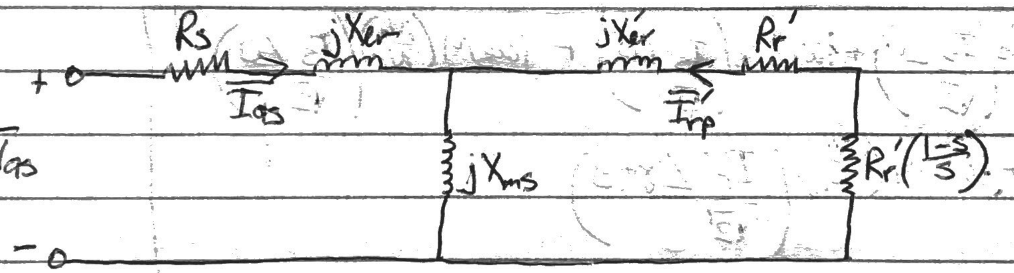
$X_{ms} = \omega_s \left(\frac{N_s}{N_r}\right) M$

$X_{lr}' = \omega_s L_{lr}'$



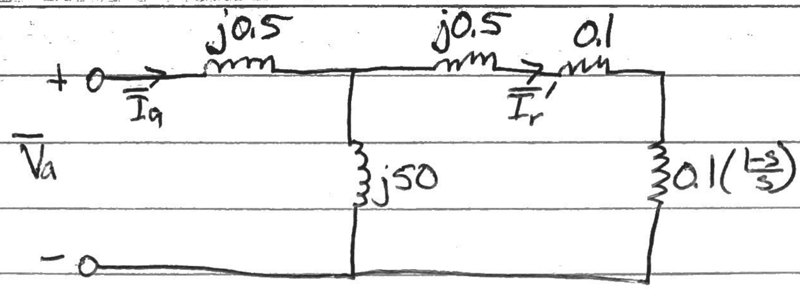
* Two types of machines:
  1) Wound Rotor: connect something to $\overline{V}_{ar}'$
  2) Squirrel cage: $\overline{V}_{ar}' = 0$

<u>For $\overline{V}_{ar}' = 0$:</u> $\frac{R_r'}{s} = R_r' + R_r'\left(\frac{1-s}{s}\right)$



* $\overline{I}_{ar}' \neq \overline{I}_{rp}'$ (different angles)

Ex

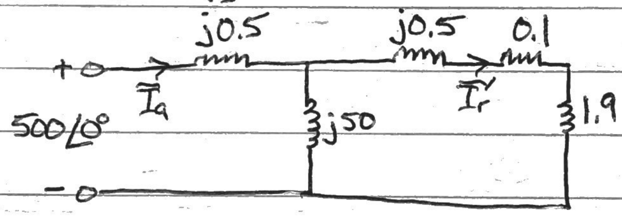


$V_L = 866\ V$ $\quad s = 0.05, \quad f = 60\ Hz$

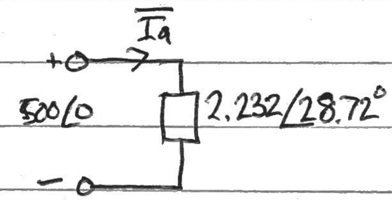
Find: a) $\bar{S}_{3\phi}$

b) $\bar{I}_r'$

Solution: $V_{an} = \frac{V_L}{\sqrt{3}} = 500V$



$\Rightarrow$



$\bar{I}_a = 224.014\angle -28.72^\circ A$

$\bar{S}_{3\phi} = 3\bar{V}_a\bar{I}_a^* \Rightarrow \bar{S}_{3\phi} = 3(500\angle 0^\circ)(224.014\angle 28.72^\circ)$

$$\boxed{\bar{S}_{3\phi} = 336.02\angle 28.72^\circ\ kVA}$$

$\bar{V}_r' = 500\angle 0^\circ - j0.5\bar{I}_a \Rightarrow \bar{V}_r' = 446.177 - j98.2277 \Rightarrow \bar{V}_r' = 456.862\angle -12.416^\circ V$

$\bar{V}_r' = (2 + j0.5)\bar{I}_r'$

$$\boxed{\bar{I}_r' = 221.606\angle -26.452^\circ A}$$